Anisotropic dust lattice modes

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Dust lattice (DL) wave modes in a one-dimensional plasma crystal suspended in the plasma sheath are studied. The ion flow in the sheath introduces an anisotropy, in particular ''ion wakes'' below the crystal particles. This leads to two types of transverse wave mode. It is shown that the ''horizontal transverse mode'' remains independent, but the ''vertical transverse mode'' and the longitudinal mode are coupled due to the particle-wake interaction. The coupling can drive an instability of the modes close to the point where their branches intersect. In addition, the particle-wake interaction might decrease the frequencies of the DL modes considerably.

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Dust lattice (DL) waves are produced by the oscillations of regularly spaced charged micrometer sized particles suspended in a plasma—i.e., in so-called plasma crystals [1–3], which form as a result of strong mutual Coulomb interaction. There have been many publications on experimental and theoretical investigations of the DL modes in one- and two-dimensional (1D and 2D) crystals [4–11]. Most investigations study longitudinal DL modes in 1D horizontal particle strings, because the string model is very convenient from the point of view of both theoretical analysis and experimental observation. In addition, the dispersion relation for the 2D longitudinal mode has been shown to be nearly coincident with that for the 1D mode [8,12].

Recently, the transverse DL mode was studied theoretically [9,10]. It was shown that the dispersion relation of the mode is described by optical-like characteristics. However, these studies did not take into account the inherent anisotropy of the systems that can be investigated experimentally. Plasma crystals have to be supported electrostatically against the pull of gravity. This implies a strong vertical electric field (e.g., such as those formed in the plasma sheath region), which in turn implies vertical ion flows that lead to the formation of ''wakes'' underneath the suspended microspheres [13,14]. The wake is created by the focusing of supersonic ion flow around each particle. Hence horizontal transverse waves should be different from vertical transverse waves. In this article we study longitudinal and transverse DL modes in a 1D horizontal particle string confined in a linear potential well, taking into account the interaction of particles with the wake. We demonstrate that the particle-wake interaction causes a coupling between the vertical transverse and longitudinal modes, and can cause an instability of the oscillations. It is shown also that the particle-wake interaction decreases the frequencies of the DL modes. The results can be easily generalized for the 2D case. Here we neglect the effect of the delayed charge variation that might drive an instability of the vertical oscillations [15,16].

We use a simplified model of the wake shown in Fig. 1. Each particle has a negative charge $-Q$. The excess positive charge of the wake downstream from the particle (along the Z axis) is considered to be a pointlike effective charge q located at distance δ. The particles form a 1D string with equilibrium separation Δ. The string is oriented along the horizontal X direction; the Y axis is perpendicular to the XZ plane. Assuming that perturbations of the particle positions do not affect the values of Q, q, and δ we can write the energy of interaction between an arbitrary particle in the string and a neighboring particle (together with the wake) in the following form:

$$U_{\rm pair}=\frac{Q^2}{\Delta_Q}e^{-\Delta_Q/\lambda}-\frac{qQ}{\Delta_q}e^{-\Delta_q/\lambda}, \tag{1}$$

where $\Delta_Q=\sqrt{(\Delta+x)^2+y^2+z^2}$ is the distance between the adjacent particles, $\Delta_q=\sqrt{(\Delta+x)^2+y^2+(\delta+z)^2}$ is the distance between one particle and the effective wake charge of the adjacent particle, and (x,y,z) denote the variation of the relative particle positions. We suppose that the interaction can be approximated by the screened Coulomb (Yukawa) potential with λ the screening length [17]. Note that the coupling with the wake below a particle does not result in a force on this particle [18], and thus the term corresponding to the qq interaction is omitted in Eq. (1). The force on a particle associated with the coupling energy (1) is $\mathbf{F}_{\rm pair}=-\partial U_{\rm pair}/\partial\mathbf{r}$. Taking into account the nearest-neighbor interaction only, we obtain the equation of motion for the nth particle in the string,

$$\ddot{\mathbf{r}}_n+2\gamma\dot{\mathbf{r}}_n=M^{-1}(\mathbf{F}_{\rm pair}^{n,n+1}+\mathbf{F}_{\rm pair}^{n,n-1}+\mathbf{F}_{\rm conf}^{n}). \tag{2}$$

Here γ is the damping rate due to neutral gas friction [19], M is the particle mass, $\mathbf{F}_{\rm pair}^{n,n\pm1}=\mathbf{F}_{\rm pair}(\mathbf{r}_{n\pm1}-\mathbf{r}_n)$, and $\mathbf{F}_{\rm conf}^{n}=-\partial U_{\rm conf}/\partial\mathbf{r}_n$ is the force due to the external confinement,

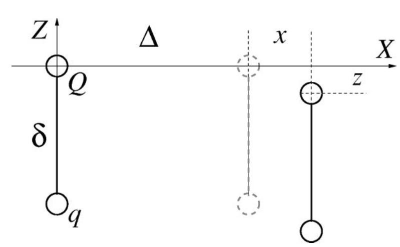

FIG. 1. Schematics of the model for the particle string with a wake. Dashed image shows the equilibrium position.

$$U_{\rm conf}=\frac{1}{2}M(\Omega_{\rm H}^2y^2+\Omega_{\rm V}^2z^2), \tag{3}$$

where $\Omega_{\rm H,V}$ are the horizontal and vertical resonance frequencies of the confinement potential, respectively. For small displacements, $|\mathbf{r}_{n\pm1}-\mathbf{r}_n|\ll\min\{\Delta,\delta\}$, we can approximately write $\Delta_Q\simeq\Delta+x$ and $\Delta_q\simeq\Delta_{q0}+(\Delta x+\delta z)/\Delta_{q0}$ (where $\Delta_{q0}=\sqrt{\Delta^2+\delta^2}$). Then we derive from Eqs. (1)–(3) the equations for the DL modes,

$$\begin{aligned}\ddot{x}_n+2\gamma\dot{x}_n&=\Omega_{xx}^2(x_{n+1}+x_{n-1}-2x_n)+\Omega_{xz}^2(z_{n+1}-z_{n-1}),\\ \ddot{y}_n+2\gamma\dot{y}_n&=-\Omega_{\rm H}^2y_n-\Omega_{yy}^2(y_{n+1}+y_{n-1}-2y_n),\\ \ddot{z}_n+2\gamma\dot{z}_n&=-\Omega_{\rm V}^2z_n-\Omega_{zz}^2(z_{n+1}+z_{n-1}-2z_n)\\ &\quad+\Omega_{zx}^2(x_{n+1}-x_{n-1}).\end{aligned} \tag{4}$$

The squared frequencies Ω_{ij}^2 are

$$\begin{aligned}\Omega_{xx}^2&=\Bigg[2+2\kappa+\kappa^2-\tilde{q}K^{-3}\\ &\quad\times\left(\frac{3+3K\kappa+K^2\kappa^2}{K^2}-1-K\kappa\right)e^{-(K-1)\kappa}\Bigg]\Omega_0^2,\\ \Omega_{yy}^2&=[1+\kappa-\tilde{q}K^{-3}(1+K\kappa)e^{-(K-1)\kappa}]\Omega_0^2,\\ \Omega_{zz}^2&=\Bigg[1+\kappa-\tilde{q}K^{-3}\\ &\quad\times\left(1+K\kappa-(K^2-1)\frac{3+3K\kappa+K^2\kappa^2}{K^2}\right)\\ &\quad\times e^{-(K-1)\kappa}\Bigg]\Omega_0^2,\\ \Omega_{xz}^2&=\Omega_{zx}^2=\tilde{q}K^{-5}\sqrt{K^2-1}(3+3K\kappa+K^2\kappa^2)e^{-(K-1)\kappa}\Omega_0^2,\end{aligned} \tag{5}$$

where $\Omega_0^2=(Q^2/M\Delta^3)e^{-\kappa}$ is the DL frequency scale, $\tilde{q}=q/Q$ is the ratio of the effective wake charge to the particle charge, $\kappa=\Delta/\lambda$ is the lattice parameter, and $K=\Delta_{q0}/\Delta\equiv\sqrt{1+(\delta/\Delta)^2}$. Normally, the interparticle separation in the plasma crystal is somewhat larger than the screening length ($\kappa\simeq1$–3), whereas the distance δ does not exceed λ (otherwise, the significant volume charge in the wake cannot exist at all). Thus, for typical experimental conditions we can set $K-1\simeq\frac{1}{2}(\delta/\Delta)^2\ll1$. Then the frequencies from Eq. (5) reduce to

$$\begin{aligned}\Omega_{xx}^2&\simeq(1-\tilde{q})(2+2\kappa+\kappa^2)\Omega_0^2\equiv\Omega_\parallel^2,\\ \Omega_{yy}^2&\simeq\Omega_{zz}^2\simeq(1-\tilde{q})(1+\kappa)\Omega_0^2\equiv\Omega_\perp^2,\\ \Omega_{xz}^2&=\Omega_{zx}^2\simeq\tilde{q}(\delta/\Delta)(3+3\kappa+\kappa^2)\Omega_0^2\equiv\Omega_{\rm coup}^2,\end{aligned} \tag{6}$$

where $\Omega_\parallel$ and $\Omega_\perp$ are the frequencies of the longitudinal (x) and transverse (y,z) DL modes, respectively, and $\Omega_{\rm coup}$ characterizes the coupling between the x and z modes induced by the particle-wake interaction. We see that the wake might decrease the main frequencies of the DL modes considerably, via the factor $(1-\tilde{q})$. Note that we neglect terms $O(\delta/\Delta)^2$ in Eq. (6).

For a traveling monochromatic wave, the particle displacements are $\mathbf{r}_n\propto\exp\{i(\omega t-nk\Delta)\}$, where k is the wave vector, $-\pi\leqslant k\Delta\leqslant\pi$. Substituting Eq. (6) in Eq. (4) we obtain the dispersion relation

$$\begin{aligned}&[\Omega_{\rm H}^2-4\Omega_\perp^2\sin^2(k\Delta/2)+2i\gamma\omega-\omega^2]\\ &\quad\times\{[4\Omega_\parallel^2\sin^2(k\Delta/2)+2i\gamma\omega-\omega^2]\\ &\quad\times[\Omega_{\rm V}^2-4\Omega_\perp^2\sin^2(k\Delta/2)+2i\gamma\omega-\omega^2]\\ &\quad+4\Omega_{\rm coup}^4\sin^2(k\Delta)\}=0.\end{aligned} \tag{7}$$

We see that in addition to the usual longitudinal DL mode, there exist two transverse modes—horizontal (y) and vertical (z). The transverse horizontal mode [corresponding to the first square bracket in Eq. (7)] is independent and has the optical-like dispersion obtained in [9]:

$$({\rm Re}\,\omega)^2\simeq\Omega_{\rm H}^2-4\Omega_\perp^2\sin^2(k\Delta/2),\quad {\rm Im}\,\omega\simeq\gamma \tag{8}$$

(here and below we assume the damping to be relatively weak, $\gamma\ll\Omega_0$). This type of mode is aperiodically unstable when $2\Omega_\perp>\Omega_{\rm H}$. Physically, this is because the particles become too compressed and the confinement is no longer sufficient to cause the linear chain to be an energetically stable configuration.

The remaining two modes—the longitudinal (x) and transverse vertical (z) [the expression in the curly brackets in Eq. (7)] are coupled. This coupling is relatively weak, since $(\Omega_{\rm coup}/\Omega_0)^2\propto\delta/\Delta\ll1$ and therefore can only be important if the branches of these modes intersect at some point (ω_0,k_0). It follows from Eq. (7) that this is possible when

$$2\sqrt{\Omega_\parallel^2+\Omega_\perp^2}>\Omega_{\rm V}. \tag{9}$$

Then the intersection point is determined by

$$\omega_0=\frac{\Omega_{\rm V}}{\sqrt{1+(\Omega_\perp/\Omega_\parallel)^2}},\quad k_0\Delta=2\arcsin\left(\frac{\omega_0}{2\Omega_\parallel}\right). \tag{10}$$

Far from the intersection point [or if the condition Eq. (9) is not satisfied and the branches do not intersect at all] the modes can be treated as independent. In this case the longitudinal (x) mode is

$${\rm Re}\,\omega\simeq2\Omega_\parallel\sin(k\Delta/2),\quad {\rm Im}\,\omega\simeq\gamma. \tag{11}$$

The coupling term results in small corrections $O(\Omega_{\rm coup}^4/\Omega_0^4)$. The transverse vertical (z) mode is

$$({\rm Re}\,\omega)^2\simeq\Omega_{\rm V}^2-4\Omega_\perp^2\sin^2(k\Delta/2),\quad {\rm Im}\,\omega\simeq\gamma. \tag{12}$$

Similar to the transverse horizontal mode [see Eq. (8)], the vertical mode is also aperiodically unstable if $2\Omega_\perp > \Omega_V$. Thus, we conclude that the linear particle chain cannot be a stable configuration when

$$2\Omega_\perp > \min\{\Omega_H, \Omega_V\}. \qquad (13)$$

In experiments the vertical resonance frequency is normally about 10–30 Hz [20,21], i.e., $\Omega_V \sim 60$–$200\ \mathrm{s}^{-1}$. The interparticle distance is of the order of the screening length and varies in the range $\Delta \sim 500-1000\ \mu$m. Then in accordance with Eq. (6) we can estimate $\Omega_\perp \sim \Omega_\parallel \sim \Omega_0 \sim 30$–$150\ \mathrm{s}^{-1}$. If $\Omega_H \sim \Omega_V$, then one can see from Eq. (13) that the instability might be easily achieved in experiments. A similar effect can also be observed in 2D plasma crystals when an increase of the particle density above a certain level results in the onset of instability [22].

Now let us consider the conditions when Eq. (9) can be satisfied, i.e., the longitudinal (11) and the vertical transverse (12) branches can intersect at the point (10). If the horizontal confinement is much weaker than the vertical one, $\Omega_H \ll \Omega_V$, then the instability condition (13) is satisfied more easily than the intersection condition (9), i.e., intersection is impossible. In the opposite case, when $\Omega_H > \Omega_V$, condition (9) is satisfied before that of Eq. (13), so that the branches can intersect. Expanding Eq. (7) around (ω_0, k_0) we obtain

$$\begin{aligned}&[\omega-\omega_0-i\gamma-\Omega_\parallel \cos(k_0\Delta/2)(k-k_0)\Delta]\\ &\quad\times[\omega-\omega_0-i\gamma+(\Omega_\perp^2/\Omega_\parallel)\cos(k_0\Delta/2)(k-k_0)\Delta]\\ &\quad=-(\Omega_{\mathrm{coup}}^2/\omega_0)^2\sin^2(k_0\Delta).\end{aligned} \qquad (14)$$

One can see from Eq. (14) that the coupling results in an instability which can exist within a narrow frequency range $|\omega-\omega_0|/\omega_0 \sim (\Omega_{\mathrm{coup}}/\Omega_V)^2 \ll 1$,

$$\mathrm{Re}\,\omega \simeq \omega_0, \quad \mathrm{Im}\,\omega \simeq \gamma - (\Omega_{\mathrm{coup}}^2/\omega_0)\sin(k_0\Delta).$$

Thus, when the neutral friction is sufficiently weak the modes become unstable. For example, if $\delta/\Delta \sim 0.3$ and $\tilde{q} \sim 0.3$, then $\Omega_{\mathrm{coup}} \sim 0.3\Omega_0 \sim 10$–$50\ \mathrm{s}^{-1}$. For a neutral gas pressure $p \sim 1$ Pa the friction coefficient $\gamma \sim 1\ \mathrm{s}^{-1}$ [19], $\omega_0 \sim \Omega_V \sim 100\ \mathrm{s}^{-1}$, and therefore the instability can be observed at $p \lesssim 1$ Pa.

In conclusion, we studied DL modes in a 1D horizontal particle string suspended in the plasma sheath, taking into account the anisotropy introduced by the interaction of particles with the wake (which is caused by ion focusing downstream of the particles). We showed that, while the horizontal transverse mode remains independent, the vertical transverse and longitudinal modes are coupled due to the particle-wake interaction. We found that this coupling can drive an instability of the modes close to the point where their branches intersect. In addition, we showed that the particle-wake interaction might decrease the frequencies of the DL modes considerably.

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